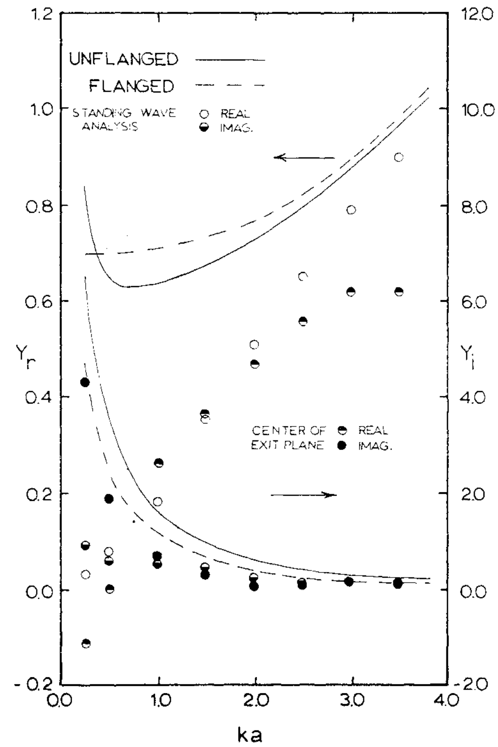

Fig. 3 Admittance at the open end of a straight duct.

Acknowledgments

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Further Observations on the Strained Coordinate Method for Transonic Flows

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Introduction

ONE of the features of a singular perturbation problem is the fact that higher approximations are more singular than the first approximation. This is due to the breakdown, in some region, of the basic assumption of small perturbations. In other words, a perturbation solution is not uniformly valid since in some region of the flow the basic assumptions do not apply. In certain classes of problems, a perturbation solution can be made uniformly valid by use of the method of matched asymptotic expansions[1] or by the method of strained coordinates.[2] In the latter method, the source of the singularity is removed by straining the independent coordinates such that the singularity apparent in the first approximation is not compounded in higher approximations. This coordinate straining is fairly arbitrary, since a number of functions can be devised that remove the compounding of the singularity.

In the last few years, a development of the strained coordinate method, as applied to transonic flow problems, has appeared in the literature.[3,4] In this method, the magnitude of the straining is determined by physical conditions (namely, that the shock wave always moves to its correct location), but there is still a considerable degree of arbitrariness concerning the choice of a straining function. In Ref. 4, an attempt is made to determine the effect of altering the form of the straining function by using two different straining functions in the computation of the same example. No discernible difference between the results was detected. However, it is desirable to prove analytically the dependence (or lack thereof) of the final pressure distribution on a particular straining function, and it is this problem that is considered here. It is found that the pressure distribution is independent of the straining function provided this function moves the shock location to the correct position.

Analysis

In a Cartesian coordinate system (x,z) with a streamwise velocity component $U(x,z)$, the total velocity for a perturbation of magnitude ϵ is given by[3]

$$U(x,z) = U_0(x',z) - \epsilon U_0(x',z)\,\delta x_s x_{1_{x'}}(x') + \epsilon U_1(x',z) \quad (1)$$

where the strained coordinate x' is related to the physical coordinate by

$$x = x' + \epsilon \delta x_s x_1(x') \quad (2)$$

and $x_1(x')$ is the straining function. Also, $U_0(x',z) = \phi_{0_{x'}}(x',z)$, and $\phi_0(x',z)$ is given by the solution of the equation

$$\phi_{0_{x'x'}} + \phi_{0_{zz}} = \phi_{0_{x'}}\phi_{0'_{x'x'}} \quad (3)$$

with the boundary condition

$$\phi_{0_z}(x',\pm 0) = \pm z_{0_{x'}}(x') \quad (4)$$

The term $U_1(x',z) = \phi_{1_{x'}}(x',z)$, and $\phi_1(x',z)$ is given by the solution of the equation

$$\phi_{1_{x'x'}} + \phi_{1_{zz}} = \left(\phi_{1_{x'}}\phi_{0_{x'}}\right)_{x'} + \delta x_s\left[x_{1_{x'}}\left(\phi_{0_{x'}} - \phi_{0_{x'}}^2\right)\right]_{x'}$$
$$+ \delta x_s x_{1_{x'}}\left(\phi_{0_{x'}} - \tfrac{1}{2}\phi_{0_{x'}}^2\right)_{x'} \quad (5)$$

with the boundary condition

$$\phi_{1_z}(x',\pm 0) = \pm z_1'(x') \pm \delta x_s x_1(x') z_0''(x') \quad (6)$$

In Eqs. (2),(5), and (6), $\epsilon\delta x_s$ is the shock movement.

In order to examine properly the behavior of the perturbation solution, it is necessary to write the equations in a common set of independent variables. On either side of the shock wave, $U(x,z)$ is piecewise continuous, and hence can be expanded in a Taylor series. Thus,

$$U(x',z) = U(x,z) - \epsilon\delta x_s x_1(x')\,U_x(x,z) \quad (7)$$

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Now,

$$U_x(x,z) \approx U_{0_{x'}}(x',z) + O(\epsilon) \tag{8}$$

and hence

$$U(x',z) \sim U(x,z) - \epsilon\delta x_s x_1(x') U_{0_x}(x',z) \tag{9}$$

Substitution of Eq. (1) into Eq. (9) gives

$$U(x',z) = U_0(x',z) + \epsilon\left[U_1(x',z) - \delta x_s\left(x_1(x')U_0\right)_{x'}\right] \tag{10}$$

This equation is valid in the continuous regions of the flow during the perturbation and should be independent of the straining function.

Equations (3) and (4) can be differentiated with respect to x' to give

$$U_{0_{x'x'}} + U_{0_{zz}} = \left(U_0 U_{0_{x'}}\right)_{x'} \tag{11}$$

and

$$U_{0_z}(x', \pm 0) = \pm z_{0_{x'x'}}(x') \tag{12}$$

Equation (11) can be multiplied by the function $\delta x_s x_1(x')$ and rearranged to give

$$\bar{U}_{0_{x'x'}} + \bar{U}_{0_{zz}} = \left(\bar{U}_{0_{x'}}\phi_{0_{x'}}\right)_{x'} + \delta x_s\left[x_{1_{x'}}\left(\phi_{0_{x'}} - \phi^2_{0_x}\right)\right]_{x'}$$

$$+ \delta x_s x_{1_{x'}}\left[\phi_{0_{x'}} - \tfrac{1}{2}\phi^2_{0_{x'}}\right]_{x'} \tag{13}$$

with the boundary condition

$$\bar{U}_{0_z}(x', \pm 0) = \pm \delta x_s x_1(x') z_{0_{x'x'}}(x') \tag{14}$$

where

$$\bar{U}_0(x',z) = \delta x_s x_1(x') U_0(x',z) \tag{15}$$

Now, Eq. (13) is identical with the linear Eq. (5) except that ϕ_1 is replaced by $\bar{U}_0$. Consequently, subtraction of these equations gives

$$\hat{U}_{0_{x'x'}} + \hat{U}_{0_{zz}} = \left(\hat{U}_{0_{x'}}\phi_{0_{x'}}\right)_{x'} \tag{16}$$

where

$$\hat{U}_0(x',z) = \phi_1(x',z) - \bar{U}_0(x',z) \tag{17}$$

The boundary condition is obtained from Eqs. (6) and (14). Thus,

$$\hat{U}_{0_z}(x', \pm 0) = \pm z_{1_{x'}}(x') \tag{18}$$

The problem uniquely defined by Eqs. (16) and (18) does not contain the straining function $x_1(x')$, and hence the solution $\hat{U}_0(x',z)$ is independent of the straining function.

Now, Eq. (10) can be rewritten as

$$U(x',z) = U_0(x',z) + \epsilon\hat{U}_{0_{x'}}(x',z) \tag{19}$$

Since neither $U_0(x',z)$ nor $\hat{U}_{0_{x'}}(x',z)$ are dependent on the straining function, it follows that the final velocity $U(x',z)$ is not dependent on the straining function.

Although Eqs. (17) and (18) can be solved without knowing the straining function, the usual formulation must be solved to give the physical coordinate x.

Acknowledgments

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Visualization of Flow Instabilities on a Rotating Disk

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Nomenclature

r = radial coordinate
R_c = cross-flow Reynolds number
R_{δ_1} = $1.271\sqrt{R_r}$
R_0 = reference radius of disk
R_r = Reynolds number = $r^2\omega/\nu$
V_c = maximum cross-flow velocity component due to secondary flow in the boundary layer (perpendicular to flow outside boundary layer)
α = nondimensional wave number
δ_1 = boundary-layer displacement thickness for the disk = $1.271\sqrt{\nu/\omega}$
ϵ = angle between the normal to the vortex axis and the radius of the disk (see Fig. 1)
λ_α = vortex spacing measured normal to vortex axis
$\bar{\lambda}_\alpha$ = λ_α/δ_1
λ_s = wavelength of secondary instabilities
λ_ϕ = vortex spacing measured in circumferential direction at a given r
ν = kinematic viscosity
ϕ = polar coordinate angle

Background

FLOW instabilities on a rotating disk have been investigated experimentally in air by Smith,[1] who used a hot-wire probe, and by Gregory et al.,[2] who used probes and also the china clay technique. Experiments were conducted in

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